

History of Maths and χ

where $\chi = \text{Gravity}$

Shape of the cosmos

A talk at the University of
Nottingham by Peter Rowlett





Watching the sky

- You will have noticed the Sun and moon moving across the sky
- Wait for long enough and you can see the stars shifting and nearby planets moving against the general background of stars

“Several billion trillion tons of superhot exploding hydrogen nuclei rose slowly above the horizon and managed to look small, cold and slightly damp.”

– Douglas Adams,
Life, the universe and everything

Watching the sky

- It is very intuitive to imagine that you are standing still and the objects you are observing are moving above you

Watching the sky

- The movement of these bodies is useful in marking the passage of time and tracking the seasons - important for growing crops
- Studying the locations of stars is useful for judging your location, which is vital for navigation

The Babylonians

- 4000 years ago
- Skilled astronomical observers
- Able to predict the apparent motion of celestial bodies
- However, there is no indication they had a model for the structure that gave rise to such apparent motion.

The Greeks

- Around 400BC, proposed a model which was developed over the following centuries, notably by Ptolemy, to become one that persisted from the 2nd Century AD for more than a thousand years

The Greeks

- Had identified Earth as spherical, certainly by around 500BC
- Since the Sun, Moon, planets and stars were moving through the Heavens, they were considered to be 'perfect' and so they were expected to orbit Earth in perfect circular motion

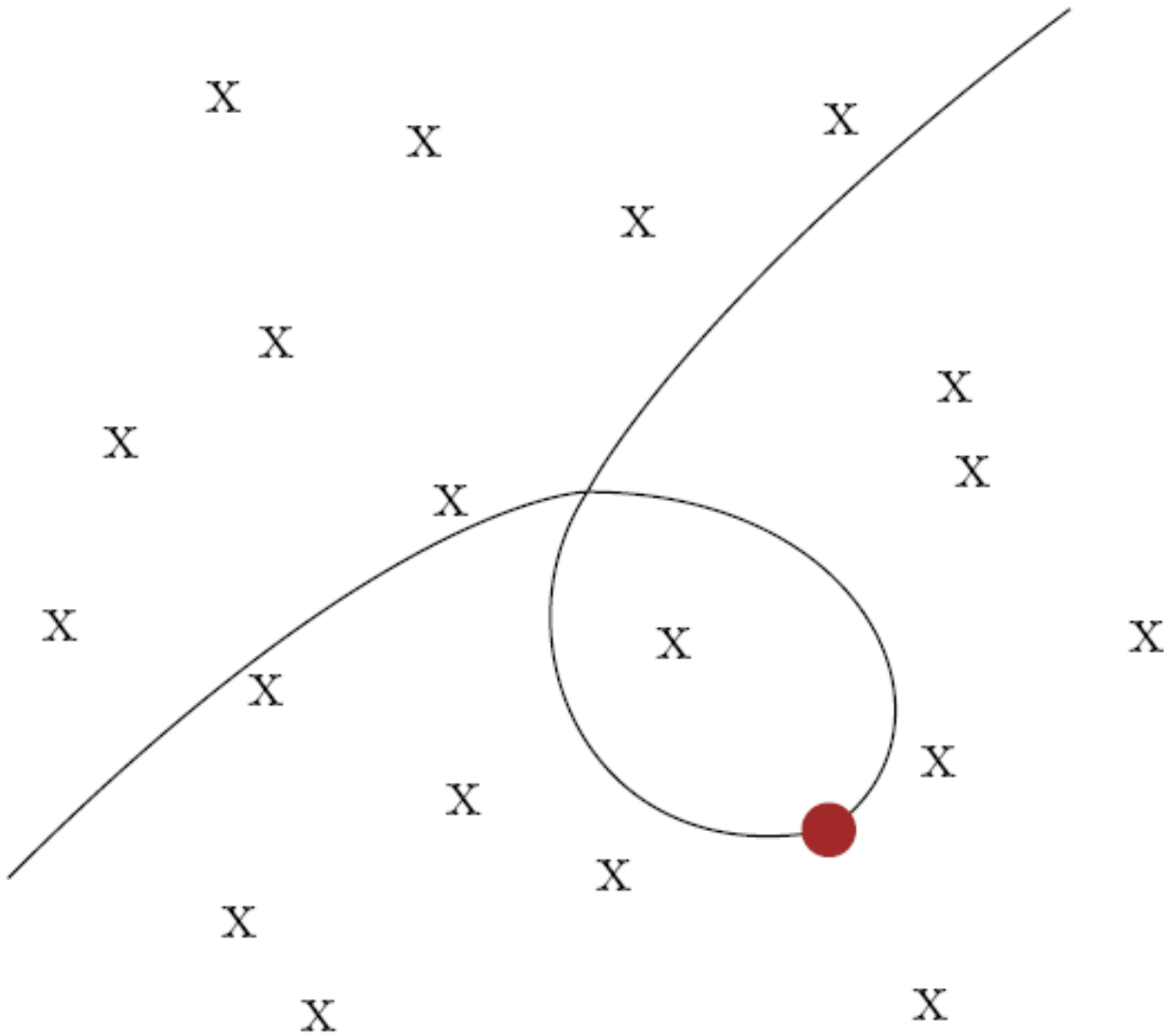
Geocentric

- A model of celestial motion with Earth at the centre
- "geo" is from a Greek word for Earth

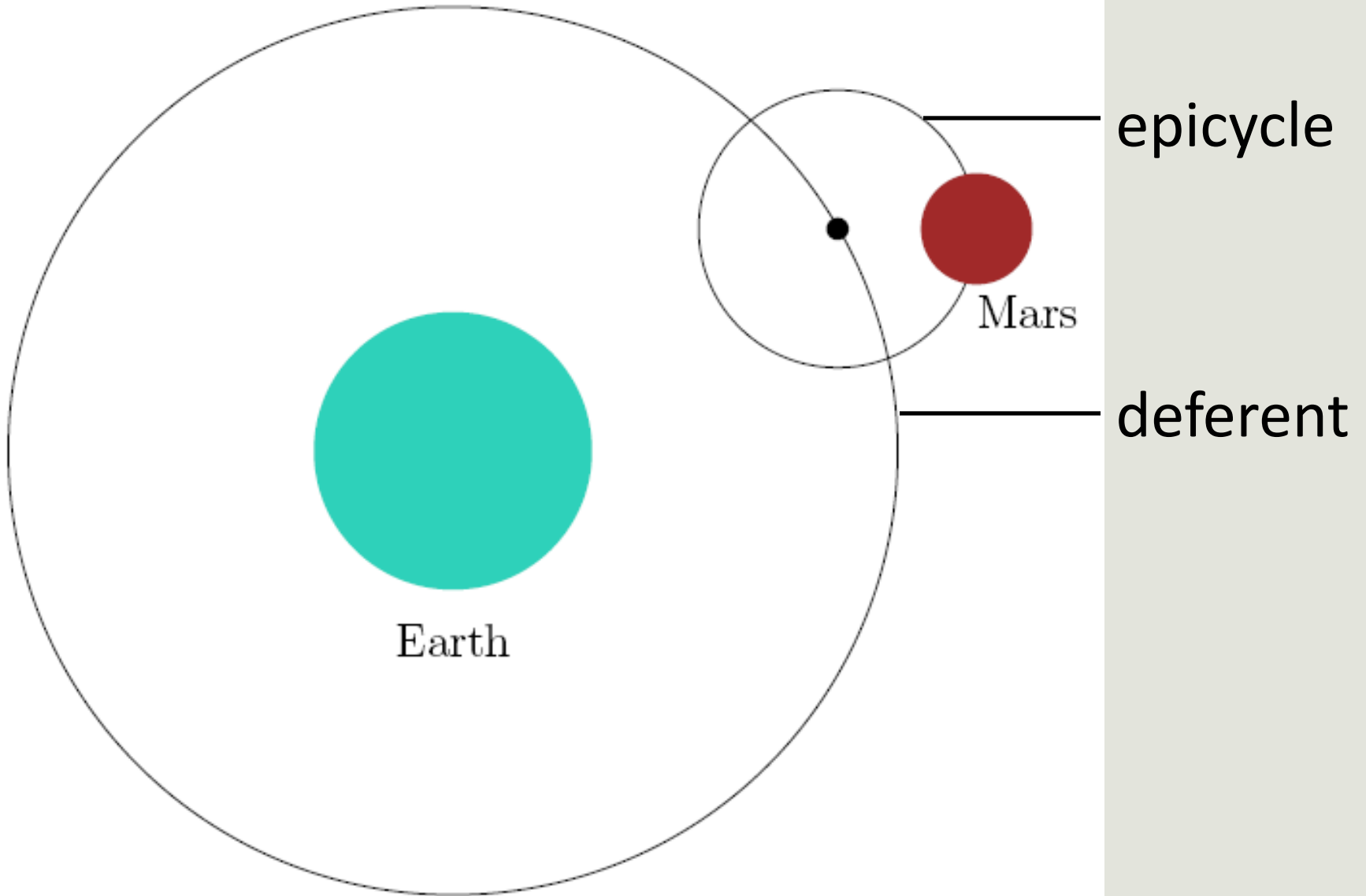
Greek model

- Viewing by naked eye, the stars appear to move as one, not shifting relative to each other as they appear to move across the sky
- This early model fixed the stars on the inside of a giant sphere which contained the Heavens
- Earth was motionless at the centre of the celestial sphere and orbited by the other known bodies, in the order: Moon, Mercury, Venus, Sun, Mars, Jupiter and Saturn

Motion of Mars against 'fixed' stars



Epicycles



Epicycles

- As time passed, increasing numbers of epicycles were used to explain all of the observed variations in speed and direction of the apparent motion of the celestial bodies

Challenging the geocentric model

- During the time this geocentric model was generally accepted, alternative models were proposed
- Particularly in the ancient Greek, ancient Indian and medieval Islamic civilisations, alternatives were proposed, including those in which Earth moves or rotates on its own axis and those with the Sun at the centre of the universe

Heliocentric

- Model with the Sun at the centre of the universe
- From the Greek "helios" for Sun

A philosophical question

- Astronomical observations could be made to fit different models
- No mathematical explanation of why an alternative model might be preferred over the established one
- Meanwhile, the Greek geocentric model was usefully predicting the movement of the heavenly bodies

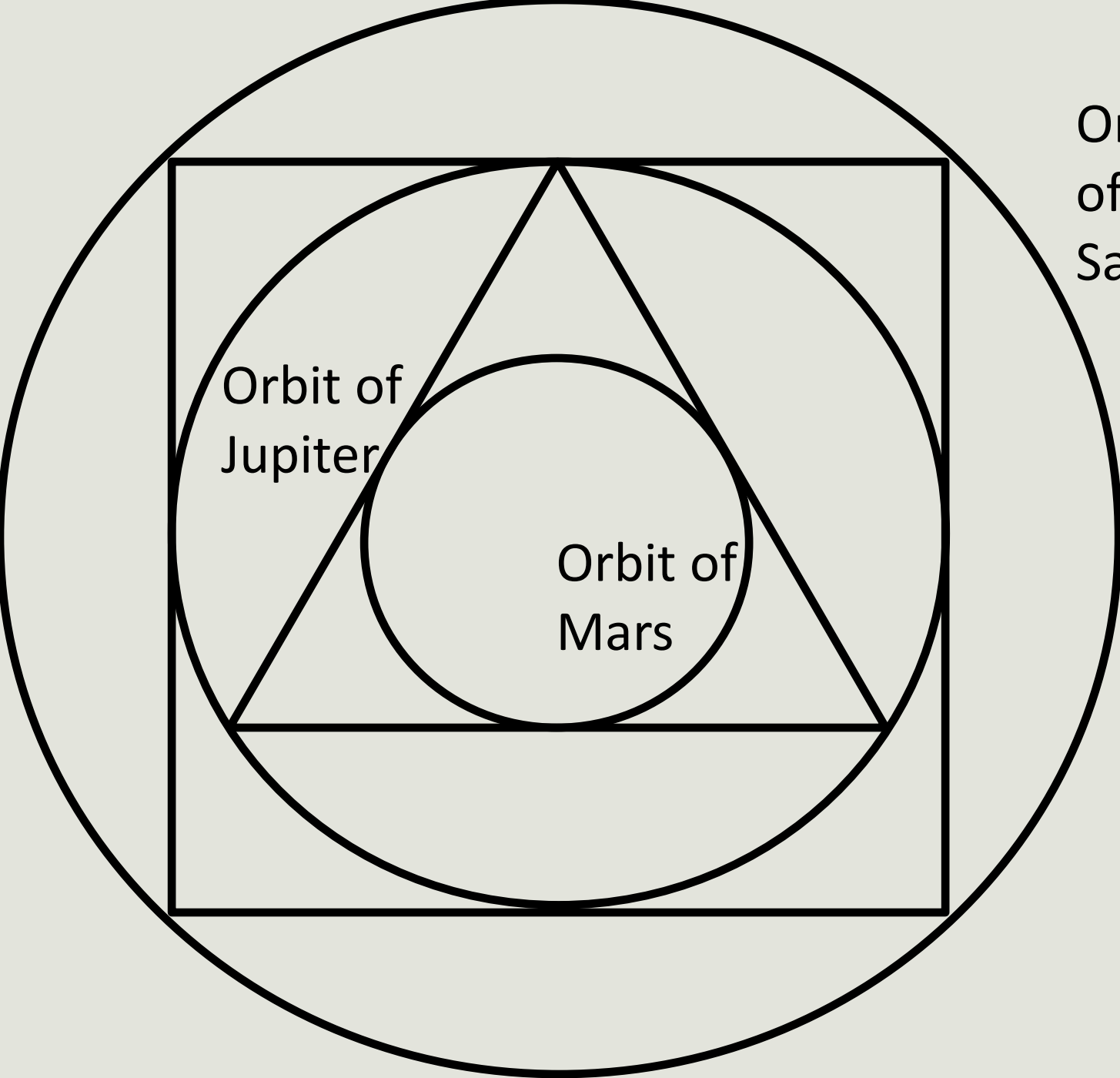
Copernicus

- At the end of his life in 1543, the capable astronomer Copernicus published a theory with the Sun at the centre of the universe and Earth and the planets orbiting the Sun
- Still based on circular motion and so, like the Greek model, used numerous epicycles to make the model fit the observed data

- The heliocentric model helps to explain some problems with a geocentric model, such as why Venus and Mercury are never seen very far from the Sun in the sky
- In the Copernican model, this is because Venus and Mercury are between Earth and the Sun
- Still, there were problems with this heliocentric model and no clear evidence to support either theory

Sizes of orbits

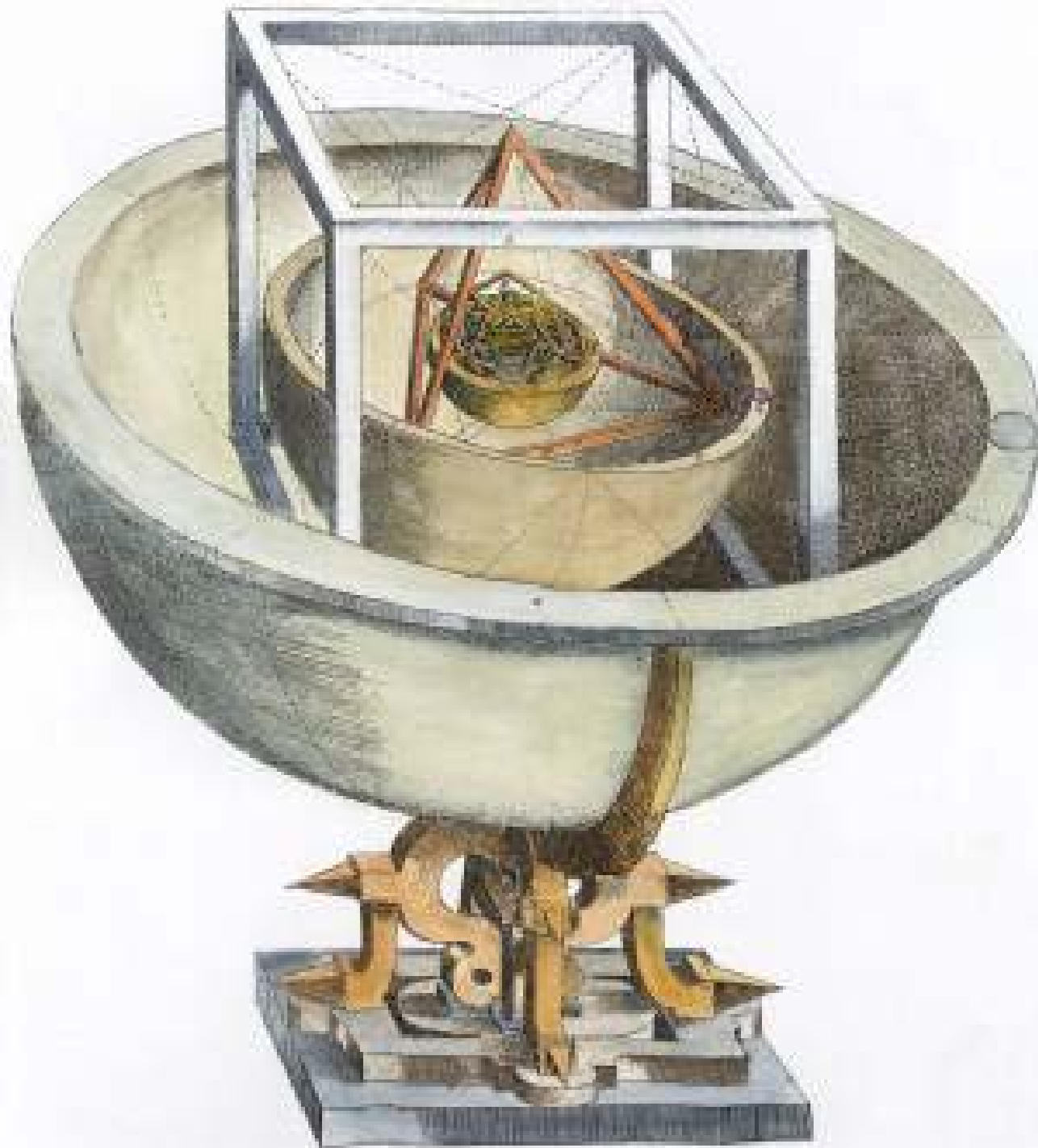
- Using the Copernican model, a calculation could be made based on observations to give the sizes of the planetary orbits
- This gave large gaps between the six planets, the reasons for which were not explained
- In the late 16th Century, Kepler proposed a heliocentric model that attempted to explain why the planets were so spaced using regular polyhedra



Orbit
of
Saturn

Orbit of
Jupiter

Orbit of
Mars



Kepler's
Platonic
model

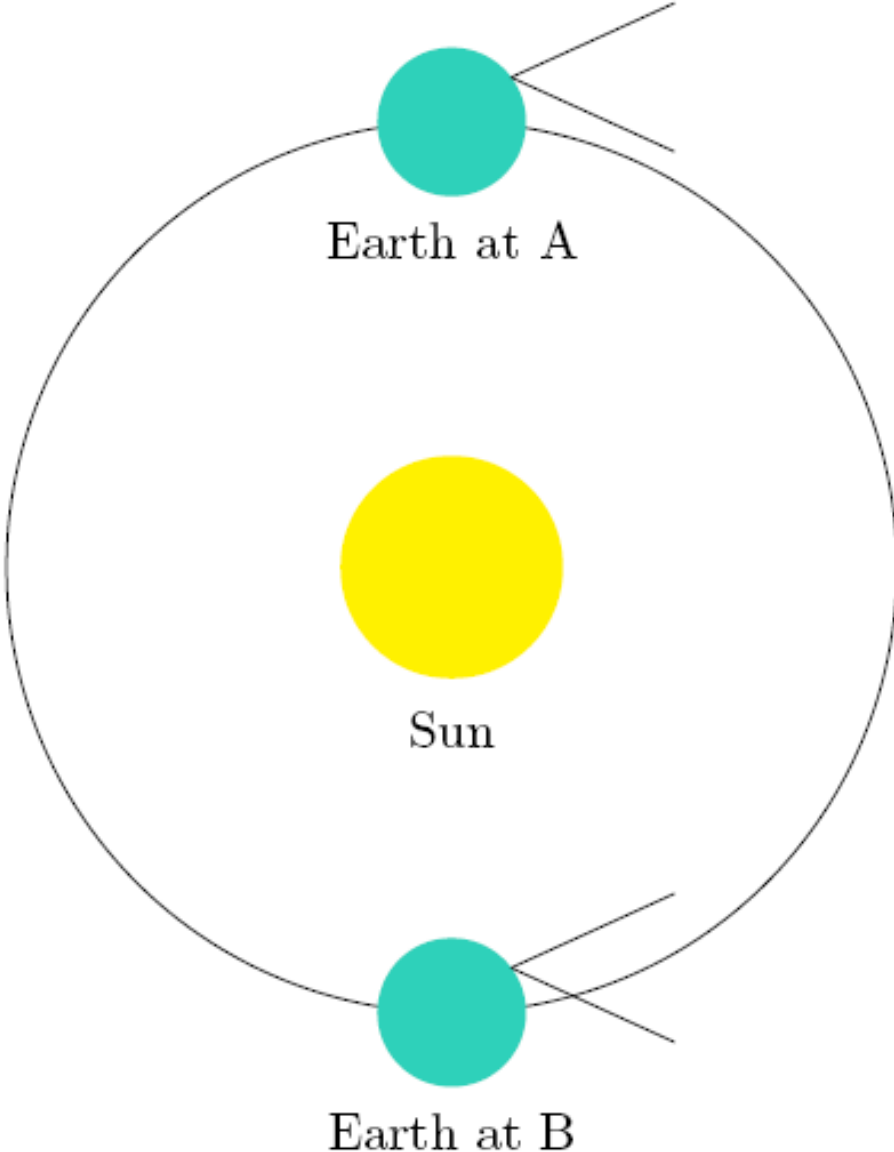
Kepler's Platonic model

- Gives an explanation for why there were at that time only 6 known planets; Euclid tells us there are precisely 5 Platonic solids
- Also fit convincingly with the sizes of the orbits of the planets estimated using the Copernican model, with the greatest error being less than 10%

Tycho Brahe

- Accomplished astronomer
- Invented new instruments and made observations during the 16th Century more accurately than those recorded previously
- Realised if Earth was orbiting the Sun, the relative positions of the fixed stars would change from different points in Earth's orbit, a phenomenon called parallax

Parallax



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Stars

Tycho's model

- Tycho's observations did not detect a parallax
- This could be explained if Earth was not moving, or if the stars were much further away than was generally thought at the time
- Instead of a heliocentric model, Tycho proposed a system with the planets orbiting the Sun and the Sun, in turn, orbiting Earth

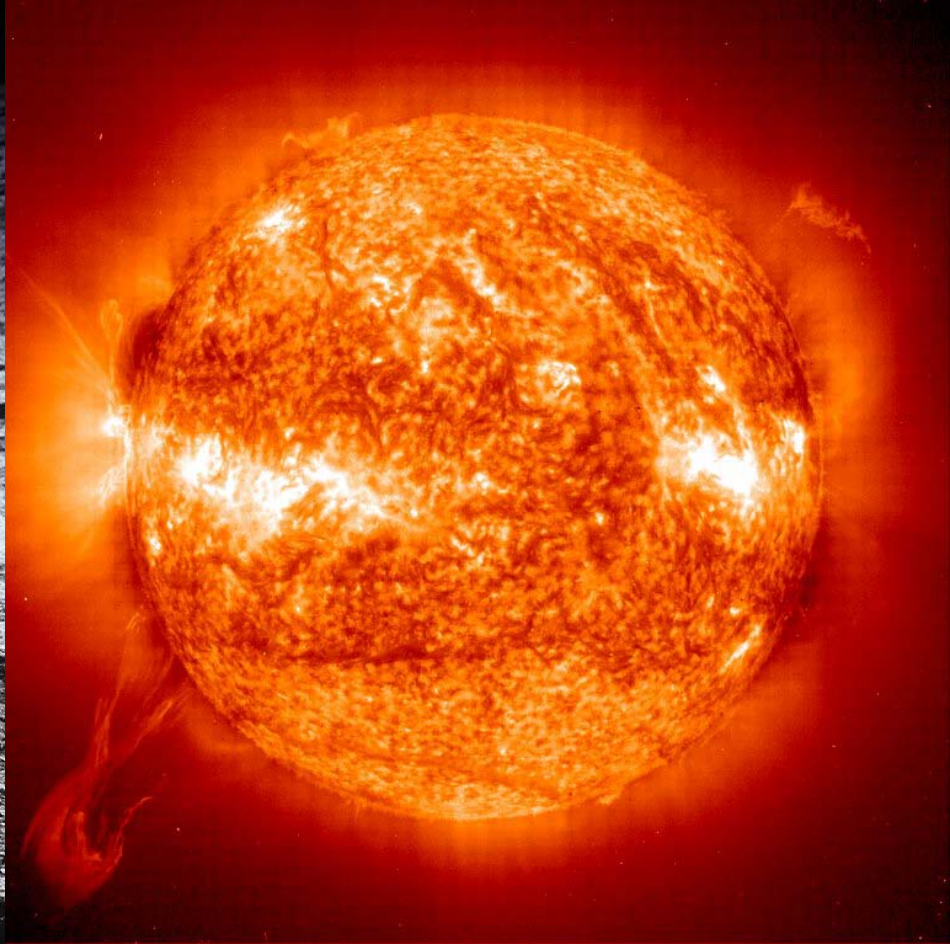
Galileo

- One of the early users of a new technology, the telescope
- Observed several moons orbiting Jupiter
 - celestial bodies not orbiting Earth
 - If Jupiter can have moons, might not at least some of the planets orbit the Sun?



Galileo

- Also published descriptions of the roughness of the moon's surface, and of sunspots on the surface of the Sun
 - If the Heavenly bodies were not perfect spheres, they might not move in perfect circles



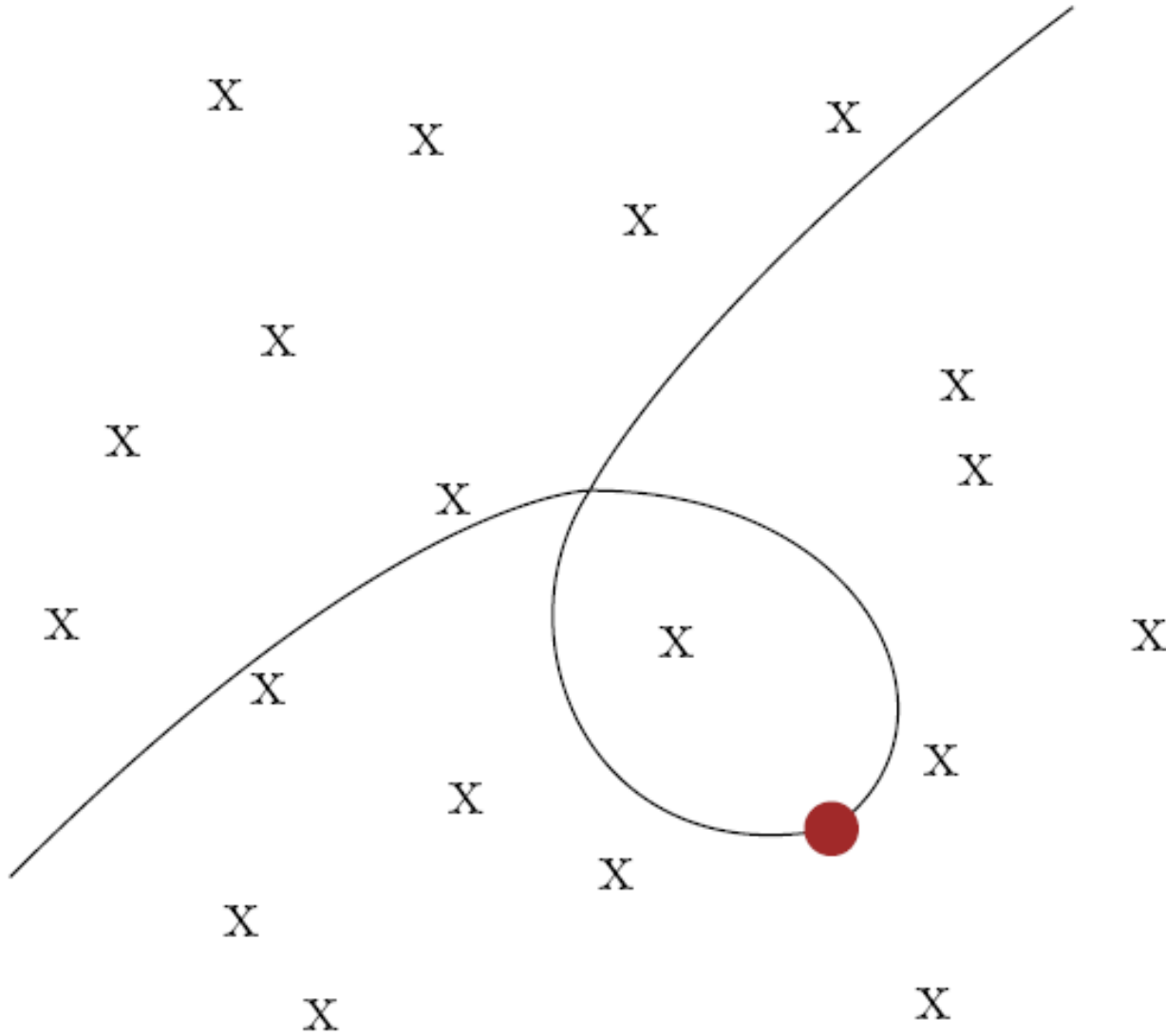
Tycho's measurements

- At the end of his life, Tycho Brahe was receiving mathematical assistance from Kepler
- On Tycho's death in 1601, Kepler inherited his measurements
- Kepler now had a large number of observations of the planets

Tycho's measurements

- Kepler attempted to use these measurements to provide evidence for a heliocentric model
- Particularly important was the position of Mars, which we have seen to have an unusual movement across the sky

Motion of Mars against 'fixed' stars



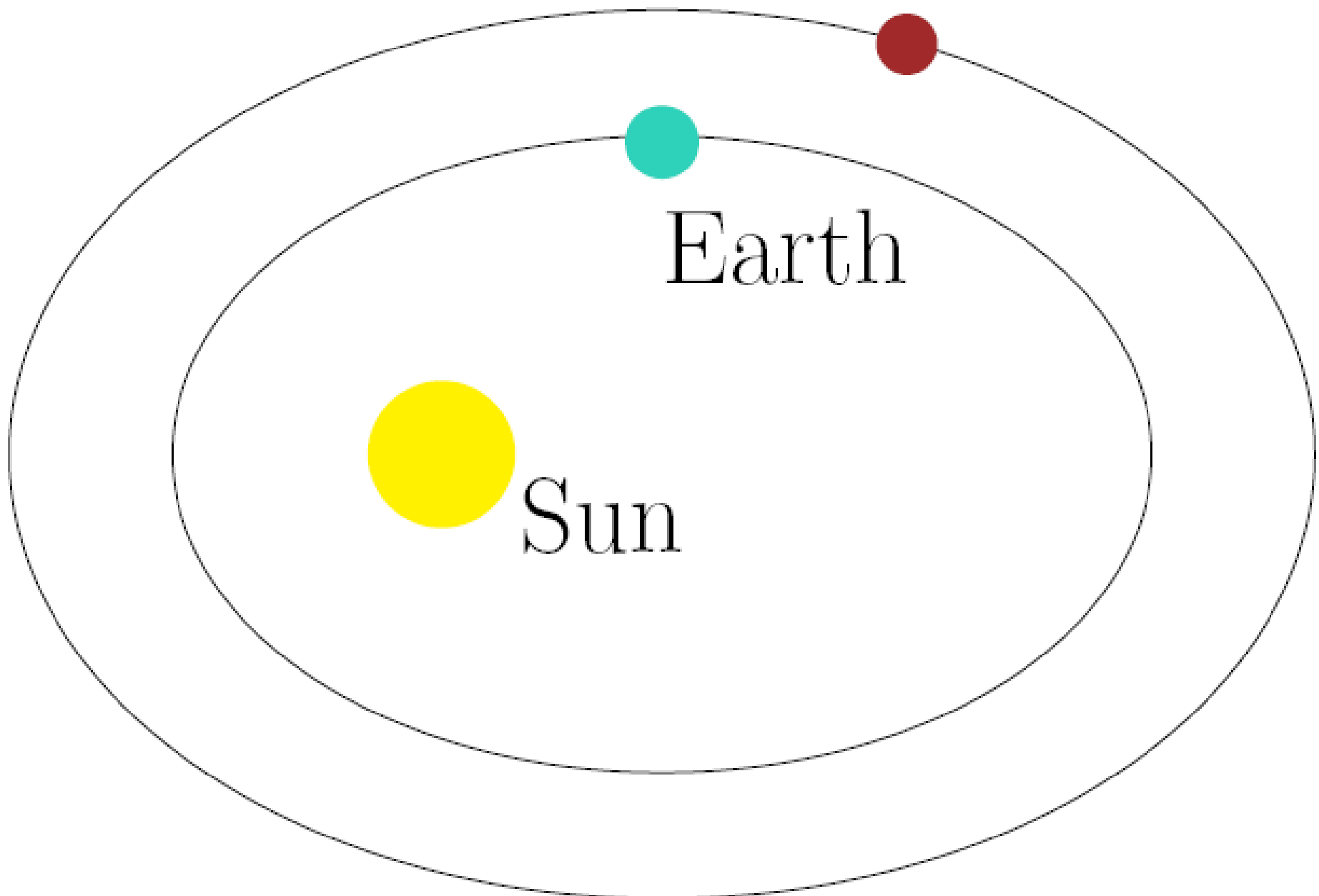
Kepler's model

- Kepler's planetary orbits are not simply paths taken by planets but also have a second independent component, representing time
- Both are described geometrically
- He calculated that rather than a heliocentric model based on epicycles, the observations would fit very well if the planetary orbits were elliptical, with the Sun at one of the foci
- Kepler produced a geometric proof of this model using Euclid's geometry

Mars

Earth

Sun



Kepler's 1st Law of Planetary Motion

The Ellipse Law - the orbit of a planet is an ellipse with the Sun at one focus of the ellipse

- broke the tradition of planetary orbits being circular

Kepler's 2nd Law of Planetary Motion

The Equal-Areas Law – the radius drawn between the planet and the Sun sweeps out equal areas in equal periods of time as the planet travels around the Sun

- recognised that the speed of a planet is not constant
- planets move more quickly when they are in the part of their orbit closer to the Sun

Kepler's 3rd Law of Planetary Motion

The Harmonic Law - The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit

- Since Mars and Earth both orbit the Sun, and Earth is closer, then Earth's orbital period is shorter than that of Mars
- In fact, a Martian year is roughly twice that of an Earth year

Kepler's 3rd Law of Planetary Motion

The Harmonic Law - The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit

- Because of the difference in the speed of their orbits, Earth sometimes catches up with and sometimes overtakes Mars
- The two orbits are not quite in the same plane
- These relative motions cause the apparent motion of Mars to appear erratic when viewed from Earth

Rudolphine Tables

- Over the following years, Kepler was able to demonstrate that observations of all 6 known planets could fit orbits that were elliptical with the Sun at a focus
- He used his theories to produce a set of astronomical predictions, the Rudolphine Tables (1627), which remained accurate for much longer than similar astronomical tables produced at the time

What causes celestial motion?

- We have avoided this philosophical question
- But the next part of this story relies on an explanation of this phenomenon

'Action at a distance'

- Aristotle says that objects do not move unless compelled to do so by an external force, and that force must act on contact between objects
- This does not allow so-called 'action at a distance', forces that act without physical contact
- Many models of celestial motion were proposed

Kepler's model

- The Sun rotates on its axis
- Rays are emitted from the rotating Sun which push the planets around in their orbits
- Planets that are further from the Sun are hit with less force and so the effect of the rays would weaken with distance

Descartes' model

- The cosmos is filled with particles of matter, called aether
- Aether moves in a series of vortices
- It is these vortices that cause the planets to move in their orbits

Which model is valid?

- Many other models were proposed
- Which model to adopt remained a matter for philosophy, since observations were not available to support a particular model and no experiment could be proposed to discover how celestial motion took place

Newton

- A force, called gravity, acts between every pair of bodies in the universe
- Connected the movement of the moon around Earth to the behaviour of objects near the Earth's surface, linking two phenomena:
 - the behaviour of a falling object when dropped
 - the motion of the celestial bodies around the cosmos
- This model is called universal gravitation

Newton's law of universal gravitation

- The force of attraction, F , exerted by two bodies on each other is proportional to their masses, m_1 and m_2 , and inversely proportional to the square of the distance, d , between them

$$F \propto \frac{m_1 m_2}{d^2}$$

The moon

- The moon is pulled towards Earth because Earth attracts it through gravity
- Earth is attracted to the moon by an equal and opposite force, but this force has less observable effect on their relationship because the mass of Earth is much greater than that of the moon

Principia

- Newton published his findings on astronomy in 1687 in his great work *Philosophiæ naturalis principia mathematica* (known as 'Principia')

Principia

- Newton shows
 - if planets are attracted to the Sun and the force of attraction acts as inverse square of the distance, Kepler's 2nd and 3rd Laws hold
 - an orbit caused by an inverse square law is a conic section (ellipse, parabola or hyperbola), which allows Kepler's 1st Law

Parallax

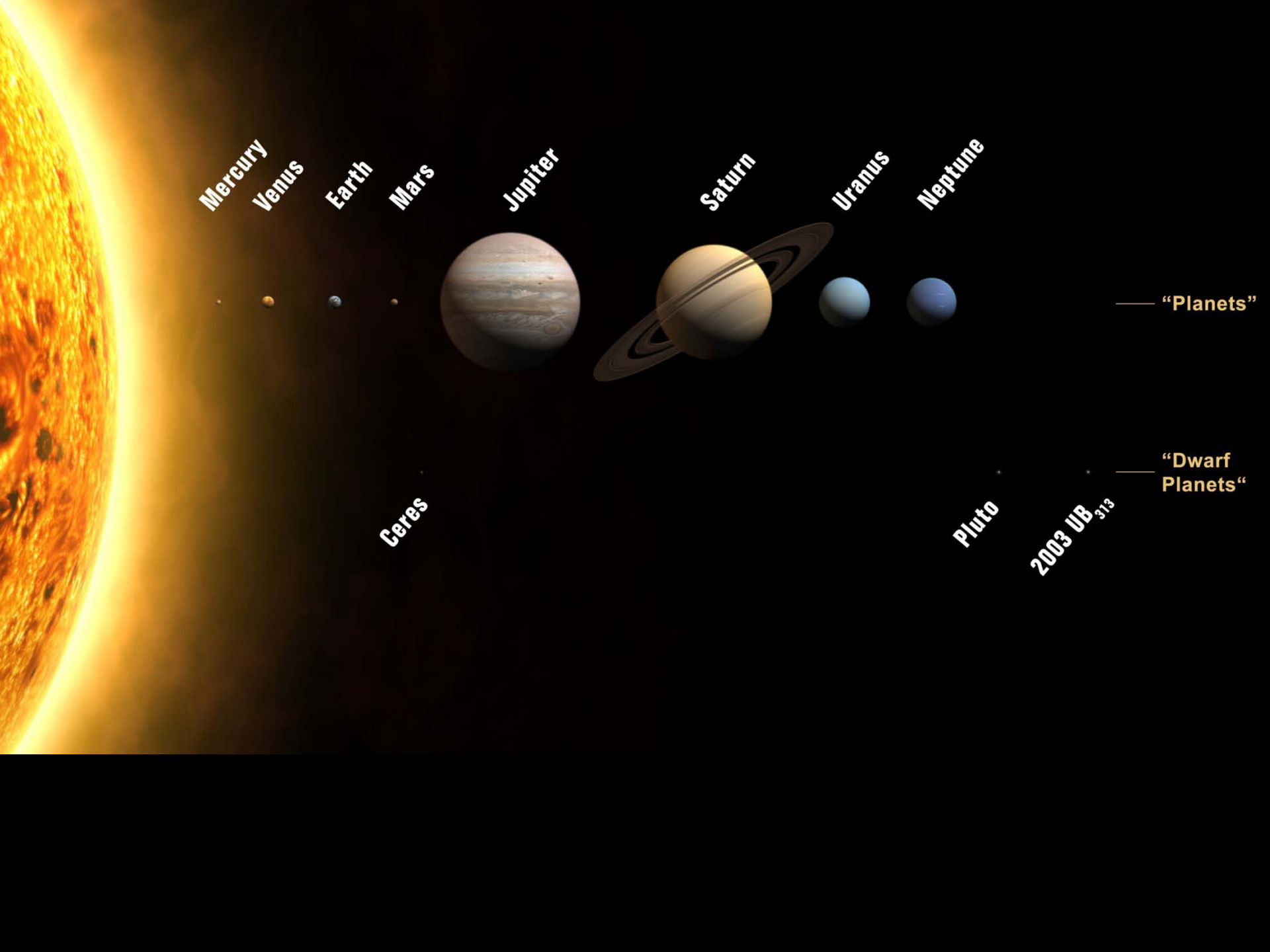
- Tycho Brahe had shown the lack of a parallax of a star provided evidence of a fixed Earth, which Kepler and Newton's heliocentric models dispute
- Parallax does occur, but is so small that it could not have been detected by Tycho's instruments

Parallax

- The first recorded measurement of the parallax of a star was in 1838 by Bessel who found a parallax for a star called 61 Cygni
- The parallax effect is smaller than expected because the stars are much further away than was thought in Tycho's time

Newtonian physics

- There were those who did not accept Newton's model, which required action at a distance
- But Newton's model predicted or explained all new discoveries in astronomy for over 200 years, after which it was ultimately replaced by a model based on Einstein's spacetime
- Our current model for the cosmos has neither Earth nor the Sun at the centre of the universe but has Earth orbiting one of many suns in part of a wider galaxy, also one of many



Mercury

Venus

Earth

Mars

Jupiter

Saturn

Uranus

Neptune

Ceres

Pluto

2003 UB₃₁₃

— “Planets”

— “Dwarf Planets”

GALAXY TRIPLET ARP 274

HUBBLE SPACE TELESCOPE ■ WFPC2



NASA, ESA, and M. Livio and the Hubble Heritage Team (STScI/AURA)

STScI-PRC09-14

Einstein

- *If you would like to find out more about the limits of Newton's model and what Einstein says about the shape of the cosmos, you can listen to the companion podcast to this talk*

Images

Space images in this presentation are from:

[flickr.com/coda](https://www.flickr.com/photos/coda/) (Night sky)

[flickr.com/dingopup](https://www.flickr.com/photos/dingopup/) (Jupiter and moons)

www.nasaimages.org (Buzz Aldrin on the moon,
sun spots, solar system, galaxy triplet)

- “Engraving of Kepler’s Platonic Universe” and “Figure of the heavenly bodies” from Wikimedia Commons
- Other pictures by Peter Rowlett

BSHM

- If you are interested in mathematical histories then you might be interested in the work of The British Society for the History of Mathematics, whose home on the web is

www.bshmm.org

History of Maths and χ

- This talk is accompanied by an audio podcast and by an article in iSquared Magazine
- You can find out how to get access to other aspects of the History of Maths and χ output through the website:

www.historyofmathsandx.co.uk